Full Length Research Paper

Optimal management fee and portfolio choice of institutional investors in a continuous-time model

Zion Guo

Department of Business Education, College of Technology, National Changhua University of Education, 500, Changhua, Taiwan, R.O.C. E-mail: zionguo@cc.ncue.edu.tw. Tel: 886-4-7232105/7344. Fax: 886-4-7211290.

Accepted 15 September, 2011

By stochastic dynamic programming setting, we derive a closed-form solution to optimal management fee and dynamic asset allocation strategy of institutional investors. We found that the optimal management fee is dependent upon managed assets, the rate of time preference, the relative risk aversion coefficient, the sensitivity of relative flow to performance, and the relative benchmark portfolio. There are two components, the benchmark hedge component and the myopic component, in the optimal dynamic asset allocation strategy of institutional investors. Both the benchmark hedge component and the myopic component are influenced by the sensitivity of relative flow to performance. Besides, the myopic component is consistent with common knowledge of investment decision and provides a possible solution to asset allocation puzzle.

Key words: Management fee, dynamic asset allocation, institutional investor.

INTRODUCTION

Over the past decades, the equity ownership of investment companies, bank trust departments, insurance companies, foundations, mutual funds and pension funds has grown dramatically. Institutional investors have become increasingly important as equity holders in financial markets. Standard financial theory assumed that investors directly invest their wealth in financial markets. It has become less appropriate as time progresses. Most people have dealings with some kind of financial institutions. With the dramatic increase in institutional ownership of equities over the past decades, institutional investors have received increasing attention in academics. Institutional investors receive the payments and deposits from their customers and face withdrawals from their customers, too. Both deposits and withdrawals are out of control, hence, institutional investors do bear the extra risk evokes from individual investors.

Institutional investors play no role in Arrow-Debreu model of resource allocation, where individual investors and firms interact through markets. The theory of continuous time finance provides a link from Arrow-Debreu world to the real world. Merton (1969, 1971) studies the issue by first understanding the behavior of an individual who acts as a market price-taker and seeks to maximize expected utility of end-of-period wealth and consumption of goods. Merton (1969) is the pioneer of using continuous-time modeling in financial economics by formulating the inter-temporal consumption and portfolio choice problem of an individual investor in a stochastic dynamic programming setting. Many researchers have extended Merton’s dynamic asset allocation model over the past decades (Bodie et al. 1992; Hindy et al. 1993; Kim and Omberg, 1996; Brennан et al., 1997; Sorensen, 1999; Liou and Poncet, 2001; Viceira, 2001; Xia, 2001; Brennan and Xia, 2002; Wachter, 2002; Yen and HsuKu, 2003; Bajeux-Besnainou et al., 2003; Chacko and Viceira, 2005; Benzoni et al., 2007; Garlappi et al., 2007; Kan and Zhou, 2007; Liu, 2007; HsuKu, 2007). However, all of those extended models did study the issue by first understanding the behavior of individual investors rather than institutional investors. Furthermore, most of the studies were on institutional investors, though there are some exceptions (Blake et al., 1999; Guo and Yen, 2006, 2008; and Guo, 2009) that do not determine holdings of
different assets by institutional investors.

The aim of this paper is to determine the optimal management fee and dynamic asset allocation strategy of institutional investors. Albeit dynamic asset allocation strategy has become an important part of research in financial economics over the past four decades, it is relatively known as the dynamic asset allocation strategy of institutional investors. Based on traditional von Neumann-Morgenstern expected utility, we derive a closed-form solution of institutional investors’ optimal management fee and dynamic asset allocation strategy.

In our model, the optimal management fee is proportional with total managed assets, the rate of time preference, relative risk aversion coefficient, sensitivity of relative flow to performance, and the relative benchmark portfolio. We found that the optimal dynamic asset allocation strategy of the institutional investor contains two components: the benchmark hedge component and the myopic component. The benchmark hedge component is an affine function of the volatility of relative benchmark portfolio. It is always positive and indicates that the institutional investor takes care of the volatility of benchmark portfolio. The myopic component is consistent with common knowledge of investment decision and provides a possible solution to asset allocation puzzle. Furthermore, both the benchmark hedge component and the myopic component are influenced by the sensitivity of relative flow to performance.

**THE ECONOMY**

**Relationship between return and flow**

Previous evidence suggests that performances influence the manager selection and termination decision. Individual investors of funds delegate its productive decisions to institutional investors, who cannot usually observe these decisions directly but can infer them from the funds’ performance and invest accordingly. Academic literature documents a positive relation between returns and cash flows of managed funds. It is well documented that individual investors of managed funds chase positive performance. Individuals tend to move cash into the managed funds that had the highest return in the preceding year.

The information content of fund returns is one of the most popular topics in finance. Many authors have found that performance is an important determinant of cash flows in managed funds and several papers have presented evidence that a flow-performance relationship does exit. Academic literature documents a convex relation between returns and fund flows of open-end mutual funds. Ippolito (1992); Sirri and Tufano (1998), and Lynch and Musto (2003) all found a small positive slope in the lower region and a considerably larger slope in the higher region, which means that net investment to be much less sensitive to returns in the region of bad returns. The highly convex flow-performance relationships of open-end mutual fund imply that open-end mutual fund investors disproportionately flock to good performers but do not punish poor performers by withdrawing assets symmetrically. Guerico and Tkac (2002) found that the empirical distribution of open-end mutual fund flows appears to be asymmetric; the top 5% experience net inflows are nearly three times larger than the outflows at the bottom 5%. Chevalier and Ellison (1997) demonstrated that the shape of the flow-performance relationship for young mutual funds is quite steep and close to linear. A 1% rise in return of an average young fund is associated with about 4% increase in the fund’s annual relative flow. In contrast, old funds have a generally convex shape. This is not the same with the mutual fund client who invest only on his own behalf. Lakonishok et al. (1992) argue that pension sponsor officials, as fiduciaries, have agency problems that induce them to value manager characteristics that are easily justified to superiors or a trustee committee. Pension fund sponsors are often financial professionals and most pension sponsors rely heavily on the recommendations of consultants that implying that pension fund sponsors are more financially sophisticated than mutual fund investors. Guerico and Tkac (2002) document that the empirical distribution of pension fund flows is more symmetric than open-end mutual fund flows, the bottom 5% of pension funds actually suffers larger dollar outflows than the top 5% gains, $524 million in outflows versus $400 million in inflows. In contrast with open-end mutual funds segment, the flow-performance relation is approximately linear in the pension funds segment:

Assumption 1: The relative flow is positively linearly related to performance

The sensitivity of relative flow to performance is the measurement of the particular kind of risk. For simplicity, it is assumed that the relationship between relative flow and performance is:

\[
flow_t = \frac{FLOW_t}{TNA_t} = SFP \times (R_t - R_{BM,t})
\]

Where, \(flow_t\) is the relative flow at time \(t\); \(FLOW_t\) is the total net cash flow at time \(t\); \(TNA_t\) is the total net managed assets of the institutional investor at time \(t\); \(SFP\) is the sensitivity of relative flow to performance, \(SFP > 0\); \(R_t\) is the fund’s return at time \(t\); \(R_{BM,t}\) is the rate of return of relative benchmark portfolio at time \(t\).

Previous evidence suggests that performances influence the manager selection and termination decision. Beginning with Jensen (1968, 1969), the evaluation of managed fund performance has generated a great deal of interest in academic and a variety of evaluation techniques have been proposed and implemented. Roll
(1978) argues that performance evaluation with benchmark portfolio is likely to be sensitive to the benchmark choice. Grinblatt and Titman (1994) found that the performance of institutional investors is sensitive to the choice of benchmark portfolios. It is generally believed that performance benchmarks are important, since they help to measure the investment performance of institutional investors and provide both clients and trustees with a reference point for monitoring that performance.

Investment opportunity sets

It is assumed that there are N risky securities and one risk-free security in the market. All of these securities can be divided infinitely and the returns of each asset accrue only in the form of capital gains (no dividend payout). There are no taxes, no transaction costs and no short-sell constraints in the market. Moreover, the institutional investor is also permitted short-sell.

It is assumed that the price of the jth security at time t, \( S_{jt} \), follows Ito process that has the following differential equation:

\[
\frac{dS_{jt}}{S_{jt}} = \mu_j dt + \sigma_j dz_j
\]  

(2)

Where, \( z_j \) is a Wiener process; \( \mu_j \) is the expected instantaneous rate of return of the jth risky security at time t; \( \sigma_j \) is the standard deviation of instantaneous rate of return of the jth risky security at time t.

Let \( BM \) be the total amount of relative benchmark portfolio at time t, and the dynamics for \( BM_i \) is:

\[
\frac{d(BM_i)}{BM_i} = \mu_{BM_i} dt + \sigma_{BM_i} dz_{BM_i}
\]  

(3)

Where, \( z_{BM} \) is a Wiener process; \( \mu_{BM} \) is the expected instantaneous rate of return of relative benchmark portfolio at time t; \( \sigma_{BM} \) is the standard deviation of instantaneous rate of return of relative benchmark portfolio at time t.

Let \( B_i \) be the total amount of the risk-free security that the institutional investor holds at time t and the dynamics for \( B_i \) is:

\[
\frac{dB_i}{B_i} = r dt
\]  

(4)

Where, \( r \) is the expected instantaneous rate of return of the risk-free security.

Management fee

Benchmark is important, and so is the fee structure. Prior to 1971, there are three different forms of fee schedules in the United States:

1. Percentage management fee schedule: A percentage of fund assets.
2. Symmetric performance-based fee schedule: A percentage of fund assets plus (minus) a preset percentage of the gain (loss) in fund value measured relative to a benchmark portfolio.
3. Asymmetric performance-based fee schedule: The same as (2) except that the percentage of the gain exceeded the percentage of the loss.

For the fear that fund managers would take on too much risk when compensated with asymmetric fees, the U.S. congress and the Securities and Exchange Commission (SEC) prohibited the use of asymmetric performance-based fee schedules in 1971. More recently, the SEC has allowed asymmetric performance fees again:

Assumption 2: The management fee is linked to the fund’s size

At time t, the management fee is defined as:

\[
MF_i = f(TNA_i)
\]  

(5)

Where, \( TNA_i \) is the total net managed assets at time t.

THE MODEL

Objective function of the institutional investor

It is assumed that institutional investors aim at solving the following dynamic portfolio choice problem:

\[
Max \sum_{i=1}^{N} E_i \left[ \int_{t}^{\infty} e^{-\delta r} U(MF_i) \, dr \right]
\]  

(6)

Where, \( w_i \) is the \( N \times 1 \) vector with representative elements \( w_{ij} \); \( w_i \) is the proportion of total net managed assets that the institutional investor invests in the jth risky security at time t, \( j = 1, ..., N \); \( E_i \) is the expectation operator at time t; \( \delta \) is the rate of time preference of the institutional investor; \( U(MF_i) \), is the utility function of institutional investors at time t.

The utility function of institutional investors is defined as:

\[
U(MF_i) = \left( \frac{1}{\gamma} (MF_i)^\gamma \right)
\]  

(7)

Where, \((1+\gamma)\) is the relative risk aversion coefficient and \( \gamma > 0 \). That is a well-known strictly concave power utility function, that is \( U'(MF_i) > 0 \) and \( U''(MF_i) < 0 \).

Derivations of our model

Since institutional investors have to allocate total net managed assets into market, it is assumed that institutional investors compose a portfolio as:
\[ TNA_t = \sum_{i=1}^{N} n_i S_{it} + B_t \]  

(8)

Where, \( n_i \) is the number of shares of the \( i \)th risky security held by institutional investors at time \( t \).

The dynamic process of total net managed assets (\( TNA_t \)) is:

\[ d(TNA_t) = \begin{cases} (1 + SFP)^{\sum_{i=1}^{N} w_i (\mu_i - \delta) dt + \sigma_d dz} \times TNA_t + r \times TNA_t \times dt - MF_t \times dt & \text{if} \ \delta \neq 0 \\ SFP \times (\mu_d dt + \sigma_d dz) \times TNA_t & \text{if} \ \delta = 0 \end{cases} \]

(9)

Institutional investors aim at solving the dynamic portfolio choice problem as:

\[ \max_{MF_t, w_t} E_t \left[ \sum_{t=0}^{\infty} e^{-\rho_t} U(MF_t) \right] \]

s.t. \( TNA_t \), Equations 5, 7, 8 and 9. The Bellman equation is:

\[ 0 = \max_{MF_t, w_t} \left[ e^{\rho_t} U(MF_t) + \frac{\partial^2 U(MF_t)}{\partial (TNA_t)^2} ((1 + SFP)(w_t, \mu_t + r \times TNA_t) - MF_t) \right] \]

(10)

\[ - \frac{\partial U(TNA_t)}{\partial (TNA_t)^2} \times SFP \times \mu_d \times TNA_t + \frac{\partial^2 U(TNA_t)}{\partial (TNA_t)^2} \times (SFP \times \sigma_d)^2 \]

\[ + \frac{1}{2} \frac{\partial^2 U(TNA_t)}{\partial (TNA_t)^2} \times (w_t, \Sigma, w_t, (1 + SFP)^2(TNA_t)^2) \]

(11)

\[ - \frac{\partial^2 U(TNA_t)}{\partial (TNA_t)^2} \times SFP \times (1 + SFP)^2(TNA_t)^2 \times w_t, \sigma_d^2 \]

Where, \( w_t \) is the transpose of \( w_t \); \( \mu_t \) is the \( N \times 1 \) vector of expected instantaneous rate of excess return of risky assets at time \( t \); \( \Sigma \) is the \( N \times N \) variance-covariance matrix of instantaneous rate of return of risky assets at time \( t \); \( \alpha_t \) is the \( N \times 1 \) vector of standard deviation of instantaneous rate of return of risky assets at time \( t \). Let \( l(TNA_t, t) = e^{\rho_t} J(TNA_t, t) \), we have:

\[ l(TNA_t, t) = e^{\rho_t} J(TNA_t, t) \]

(12)

\[ = \max_{MF_t, w_t} E_t \left[ \sum_{t=0}^{\infty} e^{-\rho_t} U(MF_t) dt \right] \]

\[ = \max_{MF_t, w_t} E_t \left[ \sum_{t=0}^{\infty} e^{-\rho_t} U(MF_t, w_t) dt \right] \]

Equation 11 is independent of explicit time. Replacing \( l(TNA_t, t) \) by \( l(TNA_t) \), we can rewrite Equation 10 as:

\[ 0 = \max_{MF_t, w_t} \left[ U(MF_t) + \frac{\partial^2 U(MF_t)}{\partial (TNA_t)^2} ((1 + SFP)(w_t, \mu_t + r \times TNA_t) - MF_t) \right] \]

(13)

For notation simplicity, we define:

\[ \Phi(MF_t, w_t) = U(MF_t) + \frac{\partial^2 U(MF_t)}{\partial (TNA_t)^2} ((1 + SFP)(w_t, \mu_t + r \times TNA_t) - MF_t) \]

(14)

Then, Equation 12 can be written as:

\[ 0 = \max_{MF_t, w_t} \Phi(MF_t, w_t) \]

The first-order conditions of Equation 14 are:

\[ \Phi_{MF_t}(MF_t, w_t) = \frac{\partial U(MF_t)}{\partial (MF_t)} - \frac{\partial l(TNA_t)}{\partial (TNA_t)} = 0 \]

(15)

and

\[ \Phi_{w_t}(MF_t, w_t) = \frac{\partial l(TNA_t)}{\partial (w_t)} = 0 \]

(16)

By Equation 15, we have:

\[ \frac{\partial l(TNA_t)}{\partial (TNA_t)} = \frac{\partial U(MF_t)}{\partial (MF_t)} \]

(17)

Let \( I = l(TNA_t) \), \( I_t = \frac{\partial l(TNA_t)}{\partial (TNA_t)} \), \( I_{tt} = \frac{\partial^2 l(TNA_t)}{\partial (TNA_t)^2} \), we know that:

\[ \frac{\partial U(MF_t)}{\partial (MF_t)} = (MF_t)^{r-1} = I_t \]

(18)

Then, the following two equations can be obtained:

\[ MF_t = \left( I_t \right)^{-1} \]

(19)

\[ U(MF_t) = - \frac{1}{V} \left( I_t \right)^{r-1} \]

(20)

By Equation 16, we have:

\[ w_t = \frac{SFP}{1 + SFP} \Sigma^{-1} \sigma_d - \frac{1}{(1 + SFP) \Sigma} \mu_c \frac{I_t}{I_{tt}} \]

(21)

After replacing Equations 19, 20 and 21 into Equation 12, we have:

\[ 0 = \left( \frac{1}{V} \right) I_t \left( \frac{1}{1 + SFP} \right) \]

(22)
where $\sigma'$ is the transpose of $\sigma$.
Let $I = A \times (TNA)^{-1}$, in order to get a trial solution, Equation (22) can be rewritten as:

$$A^{TM} = \frac{1}{1 + y} \left( -\frac{1}{y} + (1 + SFP) \mu - SFP \times \mu_{BM} + (SFP \times \sigma_{BM}) \gamma \Sigma^{-1} \mu \right) + \frac{1}{2(1 + y)} \left( -\frac{1}{y} + \frac{1}{y} + \frac{1}{2} (SFP \times \sigma_{BM}) \gamma \Sigma^{-1} \sigma \right)$$

Then, we have the optimal management fees as:

$$M_{F} = \frac{1}{1 + y} \left( -\frac{1}{y} + (1 + SFP) \mu - SFP \times \mu_{BM} + (SFP \times \sigma_{BM}) \gamma \Sigma^{-1} \mu \right) + \frac{1}{2(1 + y)} \left( -\frac{1}{y} + \frac{1}{y} + \frac{1}{2} (SFP \times \sigma_{BM}) \gamma \Sigma^{-1} \sigma \right)$$

The optimal dynamic asset allocation strategy of the institutional investor is:

$$w_t = \frac{SFP}{1 + SFP} \Sigma^{-1} \sigma_{BM} + \frac{1}{(1 + y)(1 + SFP)} \Sigma^{-1} \mu_t$$

**DISCUSSION**

The main results of the model can be generalized thus. Optimal management fee is directly proportional to total net managed assets, the sensitivity of relative flow to performance, the volatility of benchmark portfolio, and the relative risk aversion coefficient. It is inversely proportional to the rate of time preference and the expected rate of return of benchmark portfolio.

The qualitative of Equation 24 results that:

$$\frac{\partial (M_{F})}{\partial (TNA)} > 0, \frac{\partial (M_{F})}{\partial \delta} < 0, \frac{\partial (M_{F})}{\partial SFP} > 0, \frac{\partial (M_{F})}{\partial \mu_{BM}} < 0, \frac{\partial (M_{F})}{\partial \gamma} > 0, \text{and} \frac{\partial (M_{F})}{\partial \gamma} > 0$$

The first two inequalities are intuitively clear that the optimal management fee is directly proportional to total net managed assets and inversely proportional to the rate of time preference. The third one means that the increase in the sensitivity of relative flow to performance implies an increase in the management fee. The fourth inequality figures show the increase in the expected rate of return of benchmark portfolio implies a decrease in the management fee. The fifth inequality indicates that the increase in the volatility of benchmark portfolio implies an increase in the management fee. The last inequality depicts an increase in the relative risk aversion coefficient which implies also an increase in the management fee.

Optimal dynamic asset allocation strategy of institutional investors contains two components: The benchmark hedge component and the myopic component. There are two components in Equation 25. The first term is labeled “benchmark hedge component” since it is an affine function of the expected instantaneous standard deviation of relative benchmark portfolio. The position of the hedge component must be always positive. The second term is so-called “myopic component” in literature. The position of the myopic component can be either positive or negative, depending on the expected instantaneous rate of excess return.

The benchmark hedge component indicates that institutional investors take care of the volatility of benchmark portfolio. The larger the volatility of benchmark portfolio, the more the risky assets held by the institutional investor, and vice versa.

The myopic component provides a possible solution to asset allocation puzzle. The position of myopic component is inversely proportional to the relative risk aversion coefficient. This phenomenon provides a possible solution to asset allocation puzzle argued by Canner et al. (1997).^2

Both the benchmark hedge component and the myopic component are influenced by the sensitivity of relative flow to performance.

**Numerical example**

Assume one risk-free asset and three risky assets in the economy. The instantaneous rate of return of the risk-free asset is 0.01, and the excess mean vector and covariance matrix of the instantaneous rate of return of the three risky assets are:

$$\mu_t = \begin{bmatrix} 0.05 \\ 0.10 \end{bmatrix}, \Sigma_t = \begin{bmatrix} 0.0025 & 0.03 & -0.01 \\ 0.03 & 0.09 & 0.02 \\ -0.01 & 0.02 & 0.25 \end{bmatrix}$$

For simplification, it is assumed that the total net managed assets equals one and the rate of time preference of the institutional investor is equivalent to the expected instantaneous rate of return of risk-free security. Let the mean and standard deviation of the benchmark portfolio be 0.15 and 0.3, respectively.

Table 1 shows the optimal management fee and asset allocation strategy of the institutional investor as the relative risk aversion coefficient equals 0.1. In panel A, if

---

^2It is unreasonable that the management fee charged by the institutional investor will be less than zero. Hence, the variables, $\delta$ and $\gamma$, are to be set to fulfill $M_{F} \geq 0$. 

---

^3According to basic financial theory, rational investors should divide their assets between a risk-free asset and a single mutual-fund of risky assets. Risk aversion affects only the allocation between the risk-free asset and the fund. The relative proportions of different risky assets should be the same for conservative investors as for aggressive investors. In contrast to the mutual-fund separation theorem, popular financial advisors recommend that more risk-averse investors should hold a higher ratio of bonds to stocks. Canner et al. (1997) call this contradiction the "asset allocation puzzle".
Table 1. Optimal management fee and asset allocation strategy ($\gamma=0.1$).

<table>
<thead>
<tr>
<th>Panel A</th>
<th>SPF = 1</th>
<th>SPF = 2</th>
<th>SPF = 3</th>
<th>SPF = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Management fee</td>
<td>0.0035</td>
<td>0.0173</td>
<td>0.0376</td>
<td>0.0643</td>
</tr>
<tr>
<td>Risk-free asset</td>
<td>0.3078</td>
<td>-0.1037</td>
<td>-0.3094</td>
<td>-0.4329</td>
</tr>
<tr>
<td>Risky 1</td>
<td>-0.9766</td>
<td>-0.4126</td>
<td>-0.1305</td>
<td>0.0387</td>
</tr>
<tr>
<td>Risky 2</td>
<td>1.2339</td>
<td>1.0336</td>
<td>0.9335</td>
<td>0.8734</td>
</tr>
<tr>
<td>Risky 3</td>
<td>0.4349</td>
<td>0.4826</td>
<td>0.5065</td>
<td>0.5208</td>
</tr>
<tr>
<td>Expected return</td>
<td>0.1498</td>
<td>0.1651</td>
<td>0.1728</td>
<td>0.1774</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.3800</td>
<td>0.3914</td>
<td>0.3944</td>
<td>0.3953</td>
</tr>
</tbody>
</table>

| Panel B: The benchmark hedge component. |
| Risky 1 | 0.3578 | 0.4771 | 0.5367 | 0.5725 |
| Risky 2 | 0.3165 | 0.4220 | 0.4748 | 0.5064 |
| Risky 3 | 0.2890 | 0.3853 | 0.4335 | 0.4624 |

| Panel C: The myopic component. |
| Risky 1 | -1.3344 | -0.8896 | -0.6672 | -0.5338 |
| Risky 2 | 0.9174 | 0.6116 | 0.4587 | 0.3670 |
| Risky 3 | 0.1460 | 0.0973 | 0.0730 | 0.0584 |

Table 2. Optimal management fee vs. benchmark portfolio ($SFP=2, \gamma=0.2$).

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma_{BM} = 0.25$</th>
<th>$\sigma_{BM} = 0.30$</th>
<th>$\sigma_{BM} = 0.35$</th>
<th>$\sigma_{BM} = 0.40$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{BM} = 0.10$</td>
<td>0.0400</td>
<td>0.0582</td>
<td>0.0778</td>
<td>0.0989</td>
</tr>
<tr>
<td>$\mu_{BM} = 0.15$</td>
<td>0.0233</td>
<td>0.0415</td>
<td>0.0661</td>
<td>0.0822</td>
</tr>
<tr>
<td>$\mu_{BM} = 0.20$</td>
<td>0.0067</td>
<td>0.0249</td>
<td>0.0445</td>
<td>0.0655</td>
</tr>
</tbody>
</table>

$SFP = 1$, the optimal management fee is 0.0035 per dollar and the institutional investor invests 30.78% of total net managed assets in the risk-free asset and 69.22% of total net managed assets in the risky assets (short 97.66% of total net managed assets in the first risky asset, long 123.39% of total net managed assets in the second risky asset, and long 43.49% of total net managed assets in the third risky asset). The mean return and standard deviation of institutional investor’s optimal portfolio is 14.98 and 38%, respectively. Besides, the optimal management fee and the expected rate of return and risk of the optimization portfolio are increasingly as the sensitivity of relative flow to performance increases.

In panel B, we find that the positions of three risky assets of the benchmark hedge component are increasingly as the sensitivity of relative flow to performance increases. On the contrary, in panel C, both of the long positions and short position of the myopic component are decreasingly as the sensitivity of relative flow to performance increases. There are the same phenomena even if we alter the relative risk aversion coefficient. Moreover, as the relative risk aversion coefficient increases, the optimal management fee also increases.

Table 2 shows the relationship between optimal management fee and the benchmark portfolio. The numbers in Table 2 are the optimal management fees (in percentage) with specific benchmark portfolio. The optimal management fee is increasing when the volatility of the benchmark portfolio increases, and it is decreasing when the expected rate of return of benchmark portfolio increases.

Conclusions

In this paper, we derive a closed-form solution to optimal management fee and dynamic asset allocation strategy of institutional investors. We found that the optimal management fee is directly proportional to total net managed assets, the sensitivity of relative flow to performance, the volatility of benchmark portfolio, and the relative risk aversion coefficient. It is inversely proportional to the rate of time preference and the expected rate of return of benchmark portfolio. Furthermore, the optimal dynamic asset allocation is composed of two components: The benchmark hedge component and the myopic component. The benchmark hedge component displays an interesting phenomenon that, the bigger the standard
deviation of relative benchmark portfolio, the more risky the assets held by the institutional investor.

In other words, the fluctuation of relative benchmark portfolio is an incentive for holding risky assets. It makes sense since the larger the fluctuation of relative benchmark portfolio, the more the probability that institutional investors will outperform or underperform the benchmark portfolio. In order to avoid falling behind a peer group, institutional investors have to take care of the volatility of benchmark portfolio and have the incentive to trade the similar stocks as each other. The second component is a well-known "myopic component" in literature. It is consistent with common knowledge of investment decision and provides a possible solution to asset allocation puzzle. Both the benchmark hedge component and the myopic component are influenced by the sensitivity of relative flow to performance.

REFERENCES

APPENDIX

\[ d(TNA_t) = \sum_{j=1}^{N} n_j \frac{dS_{jt}}{TNA_t} + dB_t - MF_t dt + d(FLOW_t) \]

\[ = \sum_{j=1}^{N} n_j \frac{dS_{jt}}{S_{jt}} TNA_t + dB_t - MF_t dt + SFP \left( \frac{dR_t - d(BM_t)}{BM_t} \right) TNA_t \]

\[ = \sum_{j=1}^{N} w_{jt} \frac{dS_{jt}}{S_{jt}} TNA_t + dB_t - MF_t dt + SFP \left( \sum_{j=1}^{N} n_j dS_{jt} + dB_t \right) - SFP \frac{d(BM_t)}{BM_t} TNA_t \]

\[ = (1 + SFP) \left( \sum_{j=1}^{N} w_{jt} (\mu_j - r) dt + \sigma_j dz_j \right) TNA_t + r \times TNA_t \times dt - MF_t dt \]

\[ - SFP (\mu_{BM} dt + \sigma_{BM} dz_{BM}) TNA_t \]

\[ E(d(TNA_t)) = (1 + SFP) (w_i \cdot \mu_i \times TNA_t + r \times TNA_t) dt - MF_t dt - SFP \times \mu_{BM} \times TNA_t dt \]

\[ E(d(TNA_t))^2 = Var(d(TNA_t)) + E^2 (d(TNA_t)) \]

\[ = Var \left( (1 + SFP) \left( \sum_{j=1}^{N} w_{jt} (\mu_j - r) dt + \sigma_j dz_j \right) TNA_t + r \times TNA_t dt \right) - MF_t dt \]

\[ - SFP \times (\mu_{BM} dt + \sigma_{BM} dz_{BM}) \times TNA_t \}

\[ = (1 + SFP)^2 (TNA_t)^2 Var \left( \sum_{j=1}^{N} w_{jt} \sigma_j dz_j \right) + (SFP \times TNA_t)^2 Var(\sigma_{BM} dz_{BM}) \]

\[ - 2 \times SFP \times (1 + SFP) \times (TNA_t)^2 \times \text{cov} \left( \sum_{j=1}^{N} w_{jt} \sigma_j dz_j , \sigma_{BM} dz_{BM} \right) \]

\[ = (1 + SFP)^2 (TNA_t)^2 w_i \cdot \Sigma_i w_i dt + (SFP \times TNA_t)^2 \sigma_{BM}^2 dt \]

\[ - 2 \times SFP \times (1 + SFP) \times (TNA_t)^2 \times w_i \cdot \sigma_i \sigma_{BM} dt \]